Unit 14
Security and Integrity

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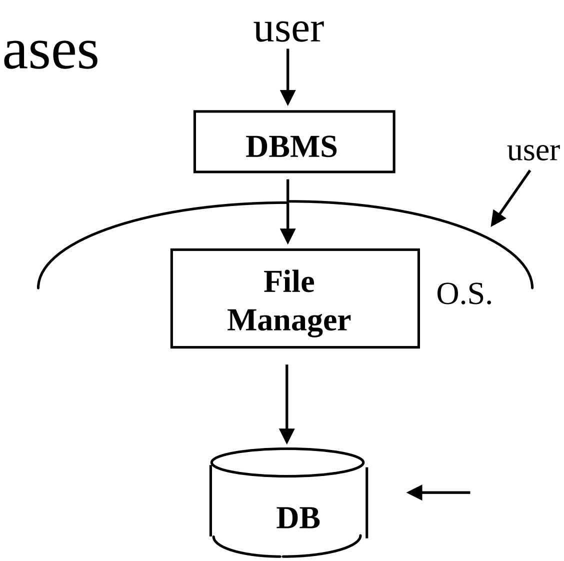


14.1 Introduction

Security and Integrity

- Objective
  - **Security**: protect data against unauthorized disclosure, alteration, or destruction.
  - **Integrity**: ensure data valid or accurate.
- Problem
  - **Security**: allowed or not ?
  - **Integrity**: correct or not ?
- Similarity between Security and Integrity
  - the system needs to be aware of <u>certain **constraints**</u> that the users must not violate.
  - **constraints** must be specified (by DBA) in some <u>**languages**</u>.
  - **constraints** must be maintained in the **system catalog** (or dictionary).
  - DBMS must monitor user interactions.

14.2 Security

General Considerations

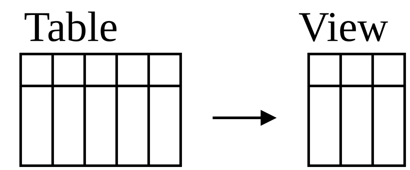
- **Aspects of the security problem:**
  - Legal, social, ethical
  - Physical control
  - O.S. security
  - **DBMS**
- **The unit of data for security purpose**
  - an entire database
  - a relation
  - a specific row-and-column data

	S.S#	S.Status	S. ...	P	SP
Charley	1	1	1 ...	0	0
	..	….	…..	.	..

- **Access Control Matrix**
  - A given user typically have different access rights on different objects.
  - Different users may have different access rights on the same object.
  - Access right on object: read, write, owner, …

Security on SQL: View Mechanism

- Two features
  - View mechanism: hide sensitive data.
  - Authorization subsystem: specify access right.



- **View Mechanism**

<e.g.1> For a user permitted access only supplier records located in Paris :

```
CREATE VIEW PARIS_SUPPLIERS
AS  SELECT S#, SNAME, STATUS, CITY
    FROM  S
    WHERE city = 'Paris';    /*value dependent*/
```

- Users of this view see a horizontal subset, similar views can be created for vertical subset or row-and-column subset.

<e.g.2> For a user permitted access to catalog entries for tables created by that user:

```
CREATE  VIEW  MY_TABLES
AS SELECT *
    FROM SYSTABLE
    WHERE CREATOR = USER; /*context dependent*/
```

Security on SQL (cont.)

<e.g.3> For a user permitted access to average shipment quantities per supplier, but not to any individual quantities :

```
CREATE  VIEW   AVQ ( S#, AVGQTY )
        AS   SELECT   S# ,  AVG(QTY)
             FROM    SP
             GROUP  BY   S#; /*statistical  summary*/
```

- **Advantages of view mechanism**
  - Provide security for free.
  - many authorization checks can be applied at compile time.

Security on SQL: Authorization Subsystem

- **In DB2, the installation procedure**
  - specify a privilege user as the system administrator.
  - the system administrator is given a special authority SYSADM, means the holder can perform every operation the system support.
  - the system administrator grant rights to other user.
  - use access control matrix

	S.S#	S.Status	S. ...	P	SP
Charley	1	1	1 ...	0	0
	..	….	…..	.	..

- **<e.g.1> [GRANT]**

```
GRANT  SELECT  ON  TABLE  S  TO  CHARLEY;
GRANT  SELECT,  UPDATE ( STATUS,  CITY )  ON  TABLE  S TO JUDY, JACK, JOHN;
GRANT  ALL  ON  TABLE  S,  P,  SP  TO FRED,  MARY;
GRANT  SELECT  ON  TABLE  P  TO  PUBLIC;
GRANT  INDEX  ON  TABLE  S  TO  PHIL;
```

- **<e.g.2> [REVOKE]**

```
REVOKE  SELECT  ON  TABLE  S  FROM  CHARLEY;
REVOKE  UPDATE  ON  TABLE  S FROM  JOHN;
REVOKE  INSERT,  DELETE  ON  TABLE  SP  FROM NANCY, JACK;
REVOKE  ALL  ON  TABLE  S,  P,  SP  FROM  SAM
```

Security on SQL: Authorization Subsystem

- **The rights that apply to tables (both base tables and views):**
  - SELECT
  - UPDATE: can specify column
  - DELETE
  - INSERT
- **The rights that apply to base tables only**
  - ALTER: right to execute ALTER TABLE
  - INDEX: right to execute CREATE INDEX
- The GRANT **option**

  User U1: GRANT SELECT ON TABLE S TO U2 WITH GRANT OPTION;

  User U2: GRANT SELECT ON TABLE S TO U3 WITH GRANT OPTION;

  User U3: GRANT SELECT ON TABLE S TO U4 WITH GRANT OPTION;

- The **REVOKE** will cascade

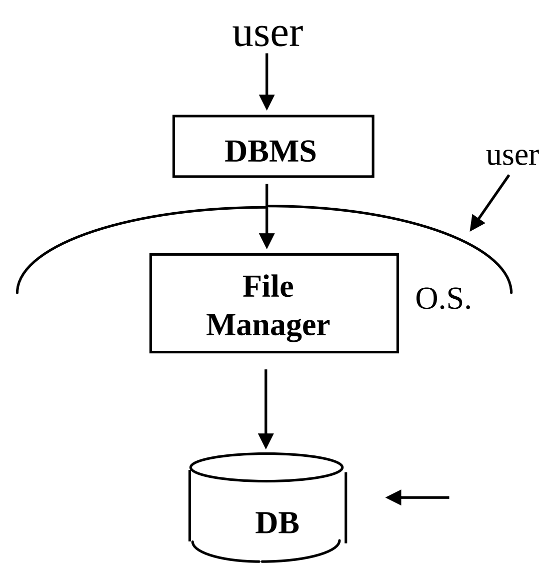
  User U1: REVOKE SELECT ON TABLE S FROM U2;
    - U3, U4, are revoked automatically!

- **Authorization** can be queried (recorded in system catalog).

Aspects of Security

- **Other Aspects of Security**
  - The total system should be secure.
  - Not to assume the security system is perfect
    - Audit Trail: keep track of all operations' information.

      <e.g.> terminal, user, date, time, ...
  - Statistical Databases: 14.5
  - Data Encryption: 14.6
  - Access Control Schemes (papers)



14.3 Integrity

General Considerations of Integrity

- **"Integrity"** refers to accuracy or correctness.
- Most of **integrity checking** today is still done by user-written procedure.
- It is preferable to specify **integrity constraints** in declarative fashion and have the system to do the check.
- Integrity constraint can be regarded as a condition that all correct states of the database required to satisfy.

  <e.g.> FORALL SX (SX.STATUS > 0)

```
CREATE  INTEGRITY  RULE  R1
   ON  INSERT   S.STATUS,
         UPDATE  S.STATUS;
   CHECK  FORALL  S ( S.STATUS > 0 )
   ELSE  REJECT;
```

- If an integrity constraint is violated, either

  <1> Reject, or

  <2> Perform **compensating action** to ensure the correctness.
- Language for specifying integrity constraints should include

  <1> the ability to specify arbitrary **conditions**.

  <2> the ability to specify **compensating actions**.

Types of Integrity Constraints

- Domain Constraints
  values of an attribute are required to belong to a pool of legal values (domain).
  <e.g.> S.STATUS > 0
  SP.QTY > 0
- Primary and Foreign key constraints
  <e.g.> S.S# must be unique
  SP.S# must be contained in S.S#.
- FD, MVD, JD
  <e.g.> S.S# => S.CITY
  $\forall$ SX $\forall$ SY (IF SX.S# = SY.S# THEN SX.CITY = SY.CITY)
- Format Constraints
  <e.g.> ID number: A999999999
- Range Constraints
  <e.g.> SALARY in (10,000 ~ 100,000).

A Hypothetical Integrity Language

- Two statements

    <1> CREATE INTEGRITY RULE
    <2> DROP INTEGRITY RULE

- <e.g.1> STATUS values must be positive:

```
CREATE  INTEGRITY  RULE  R1
   ON  INSERT    S.STATUS,
         UPDATE  S.STATUS;
   CHECK  FORALL  S ( S.STATUS > 0 )
   ELSE  REJECT;
```

    - Rule name: R1.
    - Checking times: ON INSERT S.STATUS, UPDATE S.STATUS.
    - Constraint: FORALL S (S.STATUS > 0)
    - Violation Response: REJECT.

- Default

```
CREATE  INTEGRITY  RULE  R1
   CHECK  S.STATUS > 0;
```

A Hypothetical Integrity Language (cont.)

- When a CREATE INTEGRITY RULE statement is executed:

  (1) the system check if the current database state satisfied the specified constraint,

  YES => accept the rule

  NO => reject the rule

  (2) the accepted rule is saved in **system catalog**,

  (3) DBMS monitor all operations at the specified checking times.

- The integrity rule can be dropped

  DROP INTEGRITY RULE **R1**;

A Hypothetical Integrity Language (cont.)

- <e.g.2> [The constraints can be arbitrary complex]

  Assume that **SP** includes MONTH, DAY, YEAR, each is CHAR(2), representing the date of the shipment.

```
CREATE INTEGRITY RULE R2
        CHECK IS_INTEGER (SP.YEAR)
        AND IS_INTEGER (SP.MONTH)
        AND IS_INTEGER (SP.DAY)
        AND NUM (SP.YEAR) BETWEEN 0 AND 99
        AND NUM (SP.MONTH) BETWEEN 1 AND 12
        AND NUM (SP.DAY) > 0
        AND IF NUM (SP.MONTH) IN (1,3,5,7,8,10,12)
                THEN NUM (SP.DAY) < 32
        AND IF NUM (SP.MONTH) IN (4,6,9,11)
                THEN NUM (SP.DAY) < 31
        AND IF NUM (SP.MONTH) = 2
                THEN NUM (SP.DAY) <30
        AND IF NUM (SP.MONTH) = 2
        AND NUM (SP.DAY) <>0
        AND MOD (NUM(SP.YEAR),4) = 0
                THEN NUM (SP.DAY) < 29
```

A Hypothetical Integrity Language (cont.)

- <e.g.3> Status values must never decrease

```
CREATE INTEGRITY RULE R3
    BEFORE UPDATE OF S.STATUS FROM NEW_STATUS:
    CHECK NEW_STATUS > S.STATUS;
```

- <e.g.4> The average supplier must supply greater than 25

```
CREATE INTEGRITY RULE R4
   CHECK IF EXISTS S( ) THEN AVG(S.STATUS) > 25
```

- <e.g.5> Every London supplier must supply part p2

```
CREATE INTEGRITY RULE R5
    AT COMMIT :
    CHECK IF S.CITY =  'London‘ THEN
          EXISTS SP (SP.S# = S.S# AND SP.P# = 'P2')
    ELSE ROLLBACK;
```

Note: the constraint must be checked at commit time, otherwise, it's never possible to INSERT a new S record for a supplier in 'London'.

A Hypothetical Integrity Language (cont.)

- <e.g.6> Field S# is the primary key for S.

```
CREATE INTEGRITY RULE R6
      BEFORE INSERT OF S FROM NEW_S,
              UPDATE OF S.S# FROM NEW_S.S# :
      CHECK NOT (IS_NULL(NEW_S.S#))
      AND NOT EXISTS SX ( SX.S# =NEW_S.S#)
```

Note: The syntax in SQL: PRIMARY KEY(S#) is a much better alternative.

```
CREATE TABLE S
      ( S# CHAR(20)
              .
              .
              .
        PRIMARY KEY (S#));
```

A Hypothetical Integrity Language (cont.)

- <e.g.7> S# is a foreign key in SP, matching the primary key of S.

CREATE INTEGRITY RULE **R7A**
BEFORE INSERT OF SP, UPDATE OF SP.S# :
CHECK EXISTS S (S.S#=SP.S#);

CREATE INTEGRITY RULE **R7B**
BEFORE DELETE OF S, UPDATE OF S.S# :
CHECK NOT EXISTS SP (SP.S#=S.S#);

The foreign key rule: DELETE OF S <u>RESTRICTED</u>
UPDATE OF S.S# <u>RESTRICTED</u>

A Hypothetical Integrity Language (cont.)

- The CASCADE version of foreign key rule :

```
DELETE OF S CASCADES
UPDATE OF S.S# CASCADES
```

can be represented as :

```
CREATE INTEGRITY RULE R7C
    BEFORE DELETE OF S :
    CHECK NOT EXISTS SP(SP.S#=S.S#)
    ELSE DELETE SP WHERE SP.S# =S.S#;

CREATE INTEGRITY RULE R7D
    BEFORE UPDATE OF S.S# FROM NEW_S.S#
    CHECK NOT EXISTS SP(SP.S#=S.S#)
    ELSE UPDATE SP.S# FROM NEW_S.S#
          WHERE SP.S#=S.S#;
```

<Note> The foreign key rule is more recommenced.

14.4 Security and Integrity in INGRES

Query Modification in INGRES

- <e.g.> Suppose an user U is allowed to see parts stored in London only:

DBA Permit:

DEFINE PERMIT ON P TO U
WHERE **P.CITY= "London"**

User:

RETRIEVE (P.P#, P.WEIGHT)
WHERE P.COLOR = "RED"

Automatically modified by the system

System:

RETRIEVE (P.P#, P.WEIGHT)
WHERE P.COLOR = "RED"
AND **P.CITY = "London"**

- The modification process is silent. (The user is not informed that there are other parts not located in London)
- Advantages
  - easy to implement (same as **view**).
  - comparatively efficient (security overhead occurs at query **interpretation time** rather than **execution time**).

Security Constraint in INGRES

- Syntax:

```
DEFINE PERMIT operations
        ON table [(field-
        commalist)]
        TO user
        [ AT terminal(s) ]
        [ FROM time TO time2]
        [ ON day1 TO day2 ]
        [ WHERE condition ]
```

<e.g.>

```
DEFINE PERMIT RETRIEVE, REPLACE
     ON S (SNAME, CITY)
     TO Joe
     AT TTA4
     FROM  9:00  TO  17:30
     ON SAT TO SUN
     WHERE S.STATUS < 50
           AND S.S# = SP.P#
           AND SP.P# = P.P#
           AND P.COLOR = "RED"
```

- Constraints are kept in INGRES catalog.
- The constraint identifier can be discover by querying the catalog.
- To delete a constraint

  <e.g.> DESTROY PERMIT S 27;

Integrity Constraint in INGRES

- Syntax :

```
DEFINE INTEGRITY
      ON     table
      IS     condition
```

<e.g.>

DBA:

```
DEFINE INTEGRITY
      ON    S
      IS    S.STATUS > 0
```

Suppose an user issues:

User:

```
REPLACE  S (STATUS=S.STATUS-10)
WHERE  S.CITY= "London"
```

⇩ Automatically modified by system

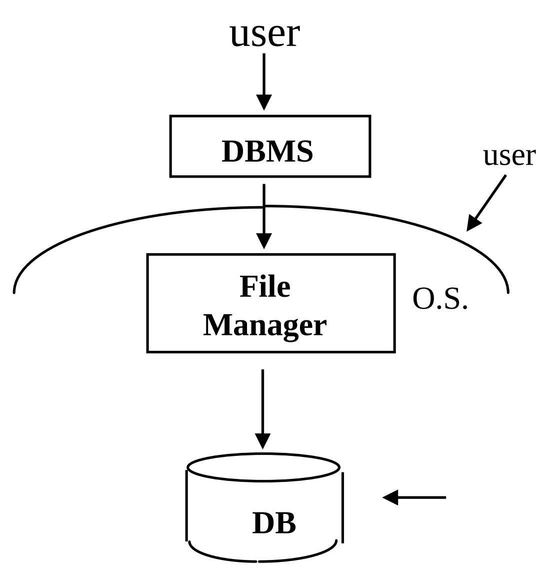
System:

```
REPLACE  S (STATUS=S.STATUS-10)
WHERE   S.CITY= "London"
        AND  (S.STATUS-10) > 0
```

- Note
  - Destroy an integrity constraint: <e.g.> DESTROY INTEGRITY **S 18**
  - The modification is silent too.
  - The integrity constraints are kept in catalog.
  - Advantages and disadvantages are similar to security constraint.

14.5 Security in Statistical Databases

- **Security**
  - The total system should be secure.
  - Not to assume the security system is perfect
  - Statistical Databases: 14.5
  - Data Encryption: 14.6
  - Access Control Schemes (papers)



Statistical Database

Def: Statistical Database is a database, such as a census database, that

(a) contains a large number of individually sensitive records .

(b) is intended to supply ~~**only**~~ statistical summary information to its users, not information to some specific individual.

- only queries that apply some **statistical function**.
  - E.g: **count**, **sum**, or **average**
- Problem:
  - "Deduction of confidential information by inference is possible"

Statistical Database: An Example

- e.g.1 Consider the **STATS** database

Name	Sex	Dependence	Occupation	Salary	Tax	Audits
Able	M	3	programmer	25k	5k	3
Baker	F	2	physician	65k	5k	0
Clark	F	0	programmer	28k	9k	1
Downs	F	2	builder	30k	6k	1
East	M	2	clerk	22k	2k	0
Ford	F	1	homemaker	51k	0k	0
Green	M	0	lawyer	95k	0k	0
Hall	M	3	homemaker	22k	1k	0
Lves	F	4	programmer	32k	5k	1
Jones	F	1	programmer	30k	10k	1

Fig. The **STATS** database

- Suppose some user **U** is intent on discovering **Able**'s salary and tax payment.
- Suppose **U** knows that **Able** is a programmer and is male

Statistical Database: Case 1

危害,信用傷害

- The security of the database has been compromised, even though U has issued only legitimate statistical quires (**count**, **sum**, or **average**.)
- **Case 1:** Q1 : SELECT **COUNT**(*)
  FROM STATS
  WHERE SEX = 'M'
  AND OCCUPATION = 'Programmer'
  Response : 1

  Q2 : SELECT **SUM** (SALARY), SUM (TAX)
  FROM STATS
  WHERE SEX = M
  AND OCCUPATION = 'Programmer'
  Response: 25k, 5k

  Solution ⇩

  The **system** should refuse to response to a query for which the cardinality of the identified subset of records < lower bound **b**

  e.g. **b** = 2 i.e., **b** ≦ **c** (result set cardinality)

Statistical Database: Case 2

Case 2: Consider the sequence of queries Q3-Q6 below

```
Q3 : SELECT   COUNT(*)
       FROM   STATS
```

Rsponse : 10

```
Q4 : SELECT   COUNT(*)
       FROM   STATS
       WHERE   NOT
                 (SEX = 'M'  AND
                  OCCUPATION = 'Programmer')
```

Response: 9
Subtract
(Q3 - Q4): 1

```
Q5 : SELECT   SUM(SALARY), SUM(TAX)
       FROM STATS
```

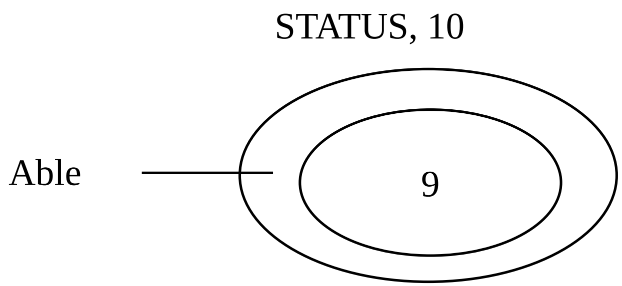
Response : 364k, 43k

```
Q6 :  SELECT   SUM(SALARY, SUM(TAX)
        FROM   STATS
        WHERE   NOT
                  (SEX = 'M'   AND
                   OCCUPATION = 'Programmer' )
```

Response 339k, 38k
Subtract
(Q5 - Q6): 25k, 5k

Statistical Database: Case 2 (cont.)

- Why ?



- Solution

  Let b ≦ c ≦ n-b eg. *2 ≦ c ≦ 8*

  Now predicate:

  NOT (Sex = 'M' and Occupation = 'programmer')

  is thus not admissible.

Statistical Database: Case 3

Case 3: Set C in the range b ≦ c ≦ n-b eg. *2 ≦ c ≦ 8* is inadequate to avoid compromise, in general.

Consider the following sequence (Q7-Q10):

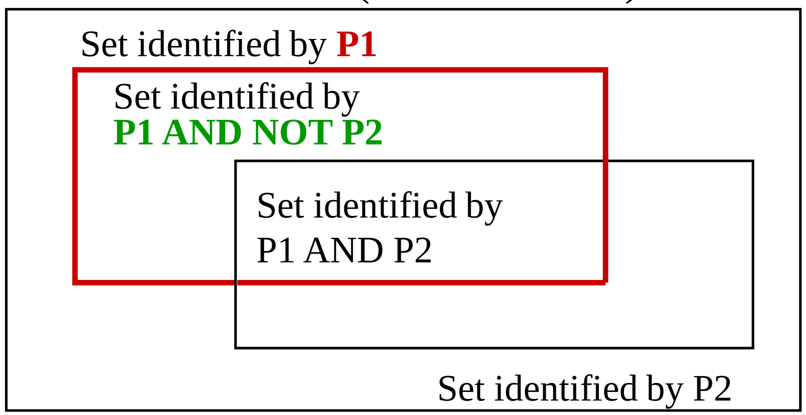
```
Q7 : SELECT   COUNT (*)
     FROM   STATS
     WHERE   SEX = 'M'                                   Response: 4

Q8 : SELECT   COUNT (*)
     FROM   STATS
     WHERE   SEX = 'M'
     AND   NOT ( OCCUPATION = 'Programmer')              Response: 3

Q9 : SELECT   SUM(SALARY), SUM(TAX)
     FROM STATS
     WHERE   SEX = 'M'                                   Response: 164k, 8k

Q10 : SELECT   SUM(SALARY), SUM(TAX)
      FROM   STATS
      WHERE   SEX = M AND NOT
           (OCCUPATION = 'Programmer')                   Response: 139k, 3k
                                                          Subtract: (Q9 - Q10): 25K, 5K
```

Statistical Database: Case 3 (cont.)

- Why ?

  P: SEX = M and OCC.= 'Programmer'
  P1: SEX = M
  P2: OCC. = 'Programmer'

Total set of records (entire database)



- Solution ???

Statistical Database: Tracker

- Individual tracker
  - for some specific inadmissible predicate
  - individual tracker, the predicate

    SEX = 'M' and NOT (OCCUPATION = 'Programmer')

    is called an **individual tracker** for Able, because it enables the user to track down information concerning Able.

- General tracker
  - is a predicate that can be used to find the answer to any inadmissible query.
  - Note : Any predicate with result set cardinality c in the range

    $$2b \leqq c \leqq n-2b$$

    where b < n/4 (typically case), is a general tracker.

Statistical Database: Case 4

Case 4:

- Assume

  1. b=2 ie. 4 $\leqq$ c $\leqq$ **6**

  2. **U** knows that **Able** is a male programmer.

     Predicate P is **SEX = 'M' and OCCUPATION = 'programmer'**

  3. **U** wishes to discover **Able**'s salary.

- Compromise steps:

  1. Make a guess at a predicate T that will serve as a general tracker,

     T: Audits = 0

  2. Find total number of individuals in the **STATS**, using **T** and **Not T**.

  3. Find the number by using P or T; P or NOT T

  4. Repeat Q11 - Q14, but using SUM instead of COUNT

  5. Able's salary: 389k- 364k = 25k

Statistical Database: An Example

- e.g.1 Consider the **STATS** database

Name	Sex	Dependence	Occupation	Salary	Tax	Audits
Able	M	3	programmer	25k	5k	3
Baker	F	2	physician	65k	5k	0
Clark	F	0	programmer	28k	9k	1
Downs	F	2	builder	30k	6k	1
East	M	2	clerk	22k	2k	0
Ford	F	1	homemaker	51k	0k	0
Green	M	0	lawyer	95k	0k	0
Hall	M	3	homemaker	22k	1k	0
Lves	F	4	programmer	32k	5k	1
Jones	F	1	programmer	30k	10k	1

Fig. The **STATS** database

- Suppose some user **U** is intent on discovering **Able**'s salary and tax payment.
- Suppose **U** knows that **Able** is a programmer and is male

Statistical Database: Case 4 (cont.).)

2. Find total number of individuals in the db, using **T** and **Not T**.

Q11 : SELECT **COUNT** (*)
FROM STATS
WHERE AUDITS = 0 Response : 5

Q12 : SELECT **COUNT** (*)
FROM STATS
WHERE NOT
(AUDITS = 0) Response : 5
Add: (Q11 + Q12) : 10

$$4 \leqq C = 5 \leqq 6$$

as a result, T is a general tracker

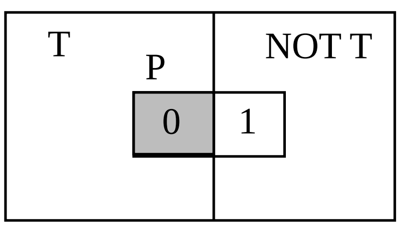
Entire database

T audits=0	NOT T
5	5

Statistical Database: Case 4 (cont.)

3. Find the number by using P or T; P or NOT T :

```
Q13 : SELECT   COUNT (*)
      FROM   STATS
      WHERE   ( SEX = 'M'   AND
                 OCCUPATION = 'Programmer')
      OR    AUDITS = 0                    Response : 6

Q14 : SELECT   COUNT (*)
      FROM   STATS
      WHERE   (SEX= 'M'   AND
                 OCCUPATION = 'Programmer')
      OR    NOT
            (AUDITS = 0 )                 Response : 5
                                          Add
                                          ( Q13 + Q14 ): 11
```



from the results we have that the number of individuals satisfying P is one; i.e., P designates Able uniquely.

Statistical Database: Case 4 (cont.))

4. Repeat Q11 - Q14, but using SUM instead of COUNT.

T or not T

Q15 : SELECT **SUM** (SALARY)
FROM STATS
WHERE AUDITS = 0 — Response : 219K

Q16 : SELECT **SUM** (SALARY)
FROM STATS
WHERE NOT (AUDITS = 0) — Response : 145K

Add (Q15 + Q16) : 364K

P or T

Q17 : SELECT **SUM** (SALARY)
FROM STATS
WHERE (SEX = 'M' AND
OCCUPATION = 'Programmer')
OR AUDITS = 0 — Response : 244K

P or not T

Q18 : SELECT **SUM** (SALARY)
FROM STATS
WHERE (SEX= 'M' ANDOCCUPATION = 'Programmer')
OR NOT (AUDITS = 0) — Response : 145K

Add (Q17 + Q18) : 389K

5. Able's salary: 389k- 364k = 25k

Statistical Database: General Tracker

- The general tracker T

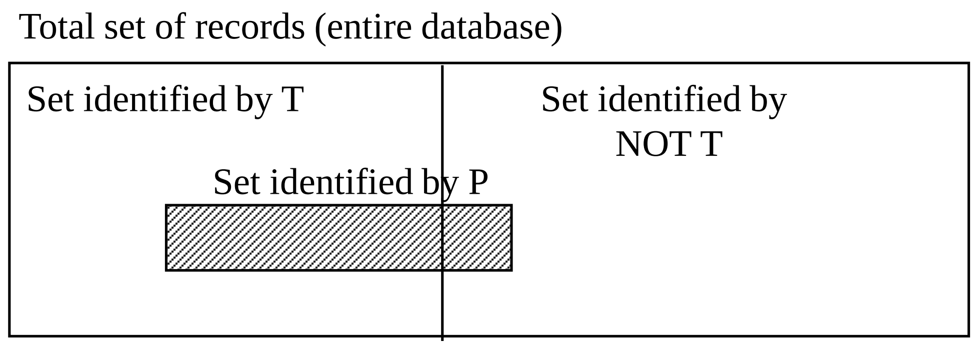


Fig. The general tracker T :

SET(P) = SET(P OR T) + SET(P OR NOT T) - SET(T OR NOT T)

- Summary
  - TODS, 4.1. D.E. Denning, et.al 79, “The Tracker: A threat to statistical database”
    - A general tracker ***almost always*** exists, and is usually both <u>easy to find</u> and <u>easy to use.</u>
    - Can be found by simply queries.
  - Security in a statistical database is a REAL PROBLEM!!

14.6 Data Encryption

Data Encryption: Basic Idea

- Infiltrator: 滲透者

  1. Using the normal system facilities for accessing the database
     - deduction information (statistical database)
     - authorization matrix
  2. Bypass the system
     - stealing a disk park
     - tapping into a communication line
     - breaks through O.S.

⇐ Data Encryption

$f(x) = 2x + 1$

231 → 463

白碼		密碼
Plaintext →	**Encryption**	→ **Ciphertext**
“**LOVE YOU**”		“**NQXGB QW**”

Encryption key: **+2**

Decryption: **-2**

Data Encryption: Basic Idea (cont.)

- e.g.2

WE NEED MORE SNOW

key = +2 →

ZG PGGF OQTG UPZY

Problem: How difficult is it for a would-be infiltrator to determine the key without prior knowledge ?

Answer: Fairly obviously, "not very" ; but equally obviously.

- e.g.3

```
   WE  NEED  MORE  SNOW
+  2 3  1 5 7 9  2 3 1 5   7 9 2 3
   YH  OJLM  PRSJ   ZWQZ
```

key = 231579

最早有關密碼的書 (1920)

Source: http://www.nsa.gov/museum/

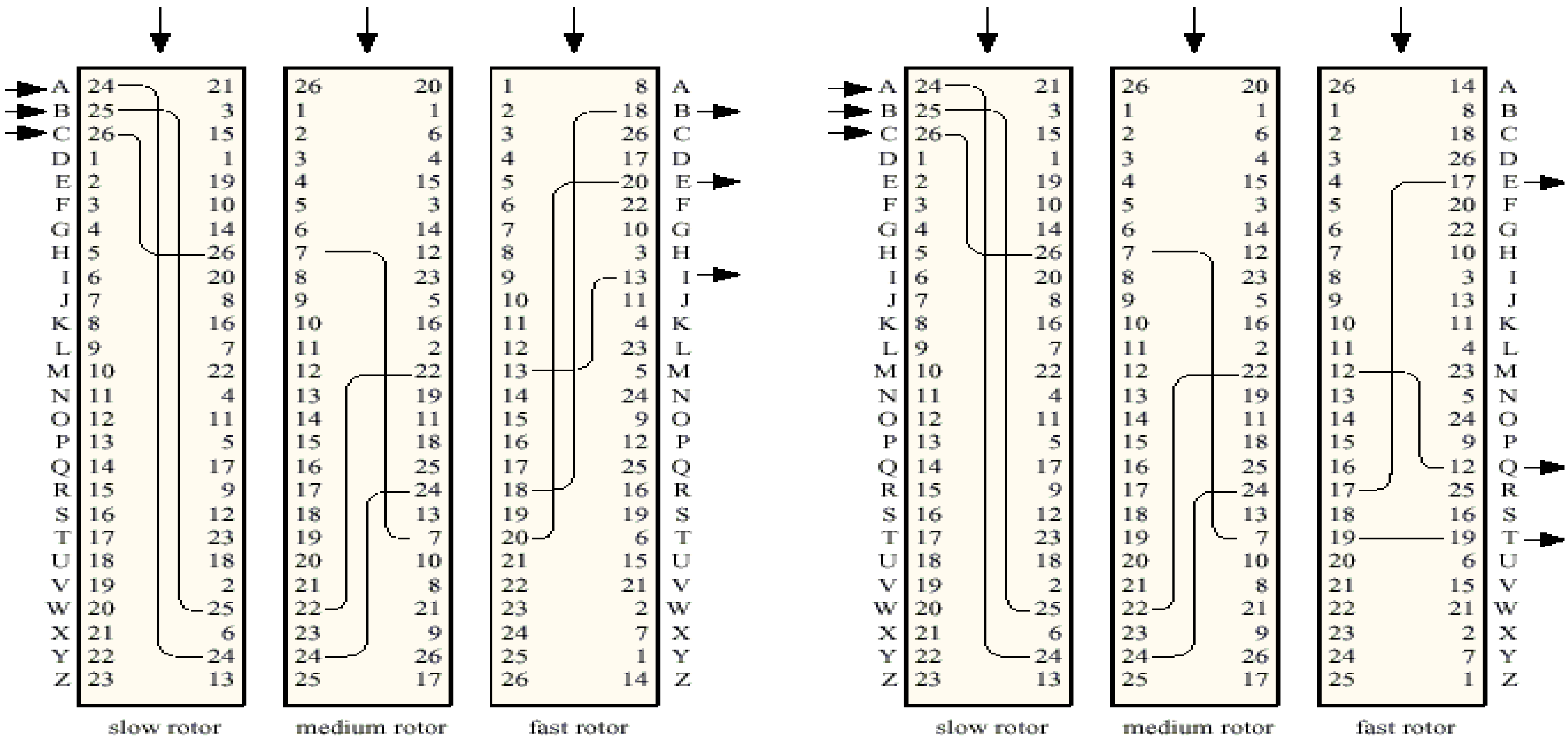
World War II

Enigma (德國)

Big machine (美國)

Source: http://www.nsa.gov/museum/

Three-Rotor Machine



(a) Initial setting

(b) Setting after one keystroke

Figure 2.8 Three-Rotor Machine With Wiring Represented by Numbered Contacts

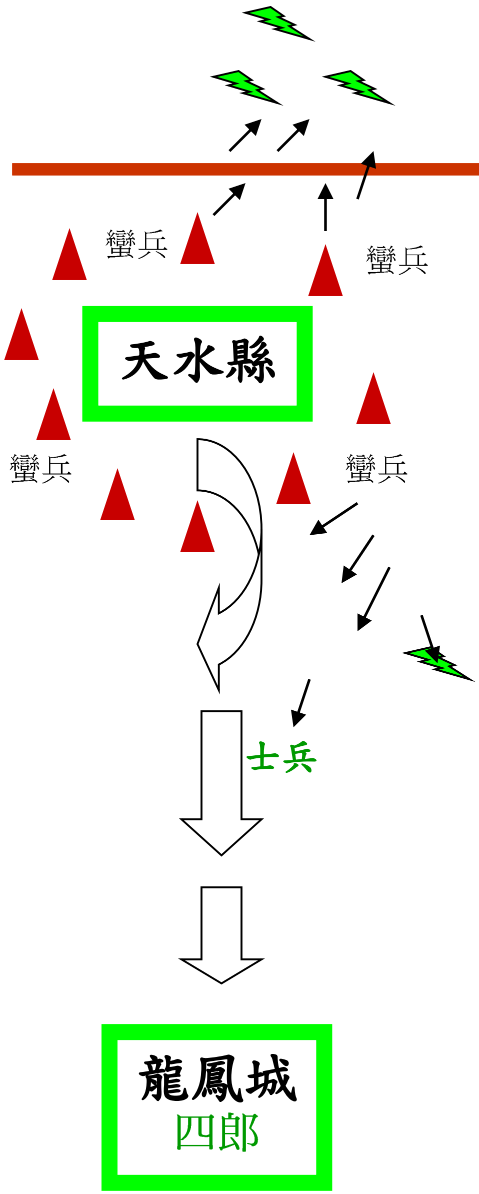
新時代

特殊IC

Supercomputer: Cray-XMP

Source: http://www.nsa.gov/museum/

諸葛四郎大鬥雙假面



四郎

資料來源: 葉宏甲編著 "諸葛四郎全集: 大鬥雙假面," 故鄉出版社, 1992.

Public Encryption: RSA 公開金鑰

- e.g.

Painttext $\boldsymbol{P} = 13$; public-key: e=11, r=15

Ciphertext $C = P^e$ modulo r

$= 13^{11}$ modulo 15

$= 1792160394037$ modulo 15

$= 7$

Decryption $\boldsymbol{P} = C^d$ modulo r

$= 7^3$ modulo15

$= 343$ modulo 15

$= 13$

甲：e, r

$\boxed{C = P^e}$

乙：e, r, d

$\boxed{P = C^d}$

Public Encryption: RSA 公開金鑰 (cont.)

- **The scheme of : [Rivest78]**

1. Choose, randomly, two distinct large primes **p** and **q**, and compute the product

$$r = p * q \qquad\qquad \text{e.g. } p=3,\ q=5,\ r=15$$

2. Choose, randomly, a large integer **e**, such that

$$\gcd(e, (p-1)*(q-1)) = 1$$

Note: any prime number greater than both p and q will do.

$$(p-1)*(q-1)= 2*4 = 8, \quad e = 11$$

3. Take the decryption key, **d**, corresponding to **e** to be the unique "multiplicative inverse" of e, modulo (p-1)*(q-1);

$$\text{i.e. } d*e = 1, \text{ modulo}(p-1)*(q- 1)$$

Note: The algorithm for computing d is straight forward.

$$d*11=1, \text{ mod } 8, \implies d=3$$

Public Encryption: RSA 公開金鑰 (cont.)

- Exercise: Suppose we have r = 2773, e = 17, try to find d = ?

Answer:

1. 50*50 = 2500 ~ 2773
2. 47*53   2773

   ≠
3. 47*59 = 2773 so p = 47, q = 59
4. (p-1)(q-1) = 2668
5. d*17 = 1, modulo (p-1)*(q-1)

   => d*17 = 1, modulo 2668

   d = 1:   17 + 2667   2668 x

   d = 2:   4 + 2667   2668 x

   ⋮

   d = 157: (157*17) / (2668) = 2

r: 50 digits => 4 hrs
75 digits => 100 days
100 digits => 74 years
500 digits => 4*10^25 years

" Signed" Ciphertext

Algorithm ENCRYPT_FOR_A

- for encryption message to be sent to A

Algorithm DECRYPT_FOR_A

- inverse of ENCRYPT_FOR_A

Algorithm ENCRYPT_FOR_B

- for encrypting message to be sent to B

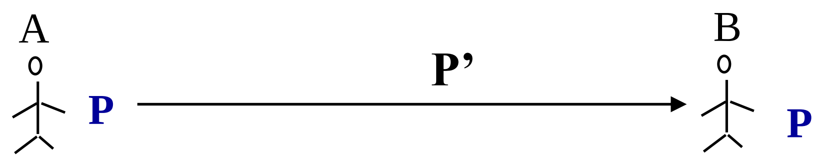
Algorithm DECRYPT_FOR_B

- ...

[A do] 1. **P'**= ENCRYPT_FOR_B(DECRYPT_FOR_A(**P**))

[sent] 2. Sent **P'** to B

[B do] 3. ENCRYPT_FOR_A (DECRYPT_FOR_B(**P'**)) => **P**



end of unit 14